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SINGULARTIES OF DYNAMIC PROCESSES PROCEEDING IN DEFORMABLE SOLIDS WITH THE FINITE RATE OF HEAT PROPAGATION TAKEN INTO ACCOUNT

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The singularities of dynamic processes occurring in deformable solids at high frequencies are studied on the basis of interrelated equations of the generalized theory of thermoelasticity.

The investigation of thermoelastic phenomena in solids has recently often been conducted on the basis of a generalized dynamical theory of thermoelasticity [1-3] with the finite rate of heat propagation in the solid taken into account. In this case the energy equation is an equation of hyperbolic type whose utilization for small times in the domain of large gradients would afford the opportunity for a more accurate description of the temperature fields [4] and the temperature stresses [2]. Experimental results on the dissipation of a heat pulse in liquid helium at very low temperatures are explained by using the hyperbolic equation of heat conduction.

In this connection, it is expedient to study the singularities of the thermoelastic processes proceeding in deformable solids by using the generalized dynamical theory of thermoelasticity.

Let us consider an infinite isotropic space possessing a thermal resistance. The thermoelastic motion of the solid can be described by the system of equations [3]

$$\mu\Delta\mathbf{u} + (\lambda+\mu)\,\mathrm{grad}\,\mathrm{div}\,\mathbf{u} - (3\lambda+2\mu)\,\alpha\,\mathrm{grad}\,(T-T_0) = \rho\,\frac{\partial^2\mathbf{u}}{\partial\tau^2},$$

$$\varkappa\Delta T = \frac{\partial T}{\partial\tau} + \tau_0\,\frac{\partial^2 T}{\partial\tau^2} + \gamma_1\left(\frac{\partial e_{ii}}{\partial\tau} + \tau_0\,\frac{\partial^2 e_{ii}}{\partial\tau^2}\right), \tag{1}$$

$$\sigma_{ij} = \lambda e_{ii}\delta_{ij} + 2\mu e_{ij} - (3\lambda+2\mu)\,\alpha\,(T-T_0)\,\delta_{ij},$$

where δ_{ij} is the Kronecker delta, Δ is the Laplace operator $\gamma_1 = (3\lambda + 2\mu)\alpha T_0/\rho c_E$.

To simplify the computations, we go over to the dimensionless variables

$$z_i = \frac{\omega^*}{c_1}\,x_i,\ \tau_1 = \omega^*\tau,\ \mathbf{u}_1 = \frac{\lambda+2\mu}{3\lambda+2\mu}\,\frac{\omega^*}{c_1}\,\frac{\mathbf{u}}{\alpha T_0},\ \theta = \frac{T-T_0}{T_0},$$

$$\Sigma_{ij} = \frac{\sigma_{ij}}{(3\lambda+2\mu)\,\alpha T_0},\ \gamma = \gamma_1\alpha\,\frac{3\lambda+2\mu}{\lambda+2\mu},\ \beta = \frac{c_1^2}{c_q^2}, \tag{2}$$

in which system (1) becomes

$$\frac{\mu}{\lambda+2\mu}\,\Delta\mathbf{u}_1 + \frac{\lambda+\mu}{\lambda+2\mu}\,\mathrm{grad}\,\mathrm{div}\,\mathbf{u}_1 - \mathrm{grad}\,\theta = \frac{\partial^2\mathbf{u}_1}{\partial\tau_1^2}, \tag{3}$$

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$$\Delta\theta = \frac{\partial\theta}{\partial\tau_1} + \beta \frac{\partial^2\theta}{\partial\tau_1^2} + \gamma\left(\frac{\partial e_{ii}}{\partial\tau_1} + \beta \frac{\partial^2 e_{ii}}{\partial\tau_1^2}\right), \tag{4}$$

$$\Sigma_{ij} = \frac{\lambda}{\lambda+2\mu} e_{ii}\delta_{ij} + \frac{2\mu}{\lambda+2\mu} e_{ij} - \theta\delta_{ij}, \tag{5}$$

where $c_1 = \sqrt{(\lambda + 2\mu)/\rho}$ is the velocity of the longitudinal elastic wave; $c_q = \sqrt{K/\rho c_E \tau_0}$, rate of propagation of the thermal wave; $\omega^* = \beta/\tau_0$, characteristic frequency of the material; and the coefficient γ characterizes the dissipation of mechanical energy.

The heat flux relaxation time τ_0 depends on the heat transmission mechanism and is sufficiently complicated to determine τ_0 exactly for specific materials, hence, mainly the order of magnitude is indicated in [3, 5]. It is known that the thermal energy in solids is transmitted by electrons and phonons. The main contribution to heat conductivity in dielectrics is from phonons, and the heat flux relaxation time in this case can be calculated from the formula $\tau_p = 3\varkappa/c_1^2$ [6]. The main contribution to heat transport in metals with their high heat conductivity is from the free electrons whose velocity is on the order of 10^6 m/sec. However, the high electron velocity still does not assure a high velocity of heat propagation. The square of the heat wave propagation velocity is directly proportional to the thermal diffusivity $\varkappa$ and inversely proportional to the heat flux relaxation time τ_0, i.e., a high rate of heat wave propagation is possible in the case of high thermal diffusivity or small relaxation time. The estimate τ_e for metals is given in [8], and the order of magnitude of τ_e is 10^{-4} sec. It is possible to compute $\tau_e = \mu^* m^*/e$ [7] approximately by means of the formula τ_e [7]. For sodium $\tau_e = 3 \cdot 10^{-14}$ sec.

Since the relaxation time ordinarily varies even because of imperfections in the configuration of the substances, then τ_0 or its associated dimensionless quantity β will later be considered a parameter in the investigation.

Let us examine the propagation of a plane harmonic wave of the form

$$\{u_1, \theta\} = \{u_0, \theta_0\} \exp(i\omega\tau_1 + pz_1). \tag{6}$$

The phase velocity and damping coefficient are determined by means of the formula

$$v_i = \frac{\omega}{\operatorname{Im} p_i}, \quad q_i = \operatorname{Re} p_i \ (i = 1,\ 2), \tag{7}$$

where $\omega = \omega_1/\omega^*$; and p_1 and p_2 are roots of the equation

$$p^4 + p^2\{\omega^2(1+\beta+\gamma\beta) - i\omega(1+\gamma)\} - i\omega^3 + \beta\omega^4 = 0, \tag{8}$$

obtained after substituting (6) into (3) and (4), and are determined by the expressions

$$\begin{gathered} p_{1,2}^2 = \frac{1}{2}\{-\omega^2(1+\beta+\gamma\beta) + i\omega(1+\gamma) \pm D_p\}, \\ D_p = \{\omega^4(1+\beta+\gamma\beta)^2 - \omega^2(1+\gamma)^2 - 2i\omega^3(1+\beta+\gamma\beta)\ (1+\gamma) - 4(-i\omega^3 + \beta\omega^4)\}^{\frac{1}{2}}. \end{gathered} \tag{9}$$

The root p_2 describes a quasielastic wave and p_1 a quasithermal wave. For high and low frequencies ω asymptotic expressions can be written for the roots p_i.

For $\omega \ll 1$, we have

$$\begin{gathered} p_1 = \pm\left[\frac{1}{2}(1+\gamma)\,\omega\right]^{\frac{1}{2}}\left\{\left(1 - \frac{\omega}{2}\left[\frac{\gamma}{(1+\gamma)^2} + \beta\right]\right) + i\left(1 + \frac{\omega}{2}\left[\frac{\gamma}{(1+\gamma)^2} + \beta\right]\right)\right\}, \\ p_2 = \pm\left[\frac{\omega^2\gamma}{2\sqrt{(1+\gamma)^5}} + i\frac{\omega}{\sqrt{1+\gamma}}\right], \end{gathered} \tag{10}$$

and for $\omega \gg 1$

$$p_1 = \pm N_1\left(\frac{M_1}{2} + i\omega\right), \quad p_2 = \pm N_2\left(\frac{M_2}{2} + i\omega\right),$$

$$N_{1,2} = \sqrt{\frac{1+\beta+\gamma\beta \mp \sqrt{(1+\beta+\gamma\beta)^2 - 4\beta}}{2}}, \quad M_{1,2} = \frac{1 + \gamma \pm \dfrac{2-(1+\beta+\gamma\beta)(1+\gamma)}{\sqrt{(1+\beta+\gamma\beta)^2-4\beta}}}{1+\beta+\gamma\beta \mp \sqrt{(1+\beta+\gamma\beta)^2-4\beta}}. \tag{11}$$

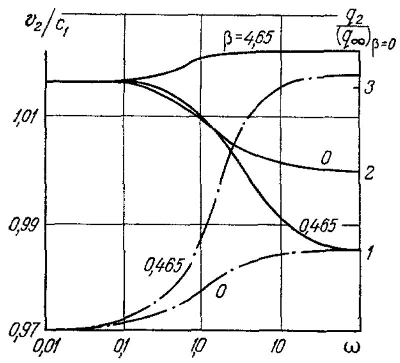

Fig. 1

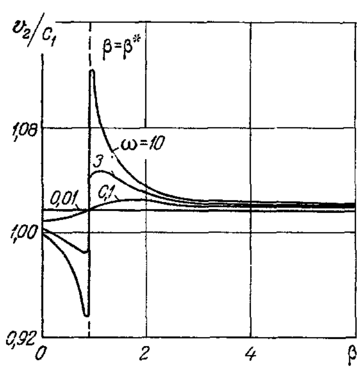

Fig. 2

Fig. 1. Dependence of the dimensionless phase velocity v_2/c_1 and damping coefficient $q_2/(q_\infty)_{\beta=0}$ on the frequency ω.

Fig. 2. Dependence of the dimensionless phase velocity v_2/c_1 on the dimensionless parameter β.

The dependence of the dimensionless phase velocity (solid lines) and damping coefficient (dash–dot lines) of a quasielastic wave on the dimensionless frequency ω is shown in Fig. 1 for different values of the parameter β with a juncture coefficient $\gamma = 0.0356$. The damping coefficient q_2 is referred to the value of the coefficient as $\omega \to \infty$ for $\beta = 0$. The dependence of the dimensionless phase velocity of a quasielastic wave on the parameter β is presented in Fig. 2 for different frequencies. Characteristics of the quasithermal wave, the dimensionless phase velocity (solid lines) and the damping coefficient (dashed lines) are illustrated in Fig. 3, where the numerical value of the damping coefficient q_1 is referred to the value of the damping coefficient as $\omega \to \infty$ for $\beta = 0.465$.

It is seen from the figures and the asymptotic expressions (10) and (11) that for the frequencies $\omega_1 \ll \omega^*$, which are ordinarily realized during mechanical vibrations, the phase velocity and damping coefficient of a quasielastic wave are practically independent of the parameter β, which means on the heat-flux relaxation time τ_0 as well; i.e., it is sufficient to use a parabolic heat conduction equation for such a frequency domain. At high frequencies $\omega_1 \gg \omega^*$, the parameter β exerts considerable influence on the characteristics of both the quasielastic and the quasithermal waves.

For $\beta < \beta^* = (1-\gamma)/(1+\gamma)^2$ the magnitude of the phase velocity of a quasielastic wave is always less than the adiabatic, while greater for $\beta > \beta^*$. Only for low frequencies ($\omega \to 0$) does it approach the value of the adiabatic velocity asymptotically in both cases. Upon passing through the point $\beta = \beta^*$, an abrupt change in the phase velocity is observed; however, this change remains finite and depends on the value of the parameter γ. In the case of the uninterconnected problem, the characteristic value β^* equals one. Thus, if

$$\tau < \tau_0^* = \frac{1-\gamma}{(1+\gamma)^2}\,\frac{1}{\omega^*}, \qquad (12)$$

then a diminution in the phase velocity with frequency is observed, but an increase for $\tau > \tau_0^*$, and τ_0^* can be considered the characteristic time for this material, which is related to the characteristic frequency and the parameter γ by relationship (12). Let us note that the frequencies of forced vibrations which can be achieved in a solid exceed the characteristic frequency by more than two orders of magnitude [9]. Values of the quantities ω^*, γ, τ_0^* are presented for certain metals in the table.

If we set $\gamma\beta = 0$, then at high frequencies the phase velocity of the quasielastic wave tends to the isothermal velocity for any values of β.

To clarify the influence of the heat-flux relaxation time on surface wave propagation, let us examine Rayleigh wave propagation in a halfspace whose boundary is stress free. Let heat exchange occur according to the Newton law between the halfspace and the surrounding medium [2]

$$\frac{\partial T}{\partial x_2} = L\,\frac{\alpha_n}{K}\,(T - T_0) \quad \text{for} \quad x_2 = 0, \qquad (13)$$

TABLE 1. Values of the Characteristic Quantities for Certain Metals

Parameters	Aluminum	Copper	Steel	Lead
ω^*, 1/sec	$4{,}66\cdot 10^{11}$	$1{,}73\cdot 10^{11}$	$1{,}75\cdot 10^{12}$	$1{,}91\cdot 10^{11}$
γ	0,0356	0,0168	0,0114	0,0733
τ_0^*, sec	$0{,}193\cdot 10^{-11}$	$0{,}549\cdot 10^{-11}$	$0{,}552\cdot 10^{-12}$	$0{,}452\cdot 10^{-11}$

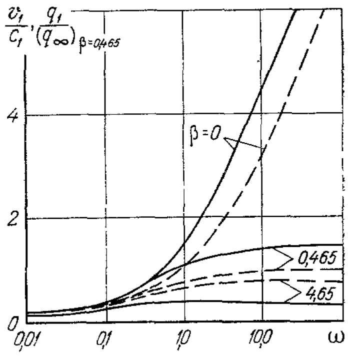

Fig. 3. Dependence of the dimensionless phase velocity v_1/c_1 and the damping coefficient $q_1/(q_1)_{\beta=0.465}$ on the frequency ω.

where α_n is the heat-transfer coefficient and $L = 1 + \tau_0\, d/d\tau$ is an operator. Since we consider harmonic wave propagation, then $L = 1 + i\omega_1\tau_0$. In this case the frequency equation has the form

$$L\left\{\left(2-\frac{v^2}{c_2^2}\right)^2\left[\frac{i(1+\gamma)}{\omega}\frac{v^2}{c_1^2}-\frac{4i}{\omega}\frac{v^2}{c_1^2}\sqrt{(1+\gamma)\left(1+\gamma-\frac{v^2}{c_1^2}\right)\left(1-\frac{v^2}{c_2^2}\right)}\right]\right\}=-\frac{\alpha_n L^{3/2}(1+i)}{Kp_R}$$

$$\times\left[\left(2-\frac{v^2}{c_2^2}\right)^2\frac{v}{c_1}\sqrt{\frac{1+\gamma}{2\omega}}+\frac{4}{\sqrt{2\omega}}\frac{v}{c_1}\sqrt{\left(1-\frac{v^2}{c_2^2}\right)\left(1+\gamma-\frac{v^2}{c_1^2}\right)}\right], \quad (14)$$

where c_2 is the velocity of shear wave propagation and p_R is the wave number. In the case of low frequencies $\omega_1 \ll \omega^*$

$$\left(2-\frac{v^2}{c_2^2}\right)^2=4\sqrt{\left(1-\frac{v^2}{c_2^2}\right)\left(1-\frac{v^2}{c_s^2}\right)},\quad c_s=c_1\sqrt{1+\gamma}, \quad (15)$$

i.e., we obtain the known relationship from the theory of thermoelastic Rayleigh waves.

To determine the influence of the parameter β on the vibrations of a rod in the resonance frequency domain, let us examine the longitudinal vibrations of a rod of length l subjected to a periodic force

$$g(z,\ \tau_1)=g_0(z)\cos\omega\tau_1 \quad (16)$$

under homogeneous boundary conditions on the displacements

$$u_1\Big|_{\substack{z=0\\z=l}}=0,\ u_1\Big|_{\tau_1=0}=0,\qquad \frac{\partial u_1}{\partial\tau_1}\Big|_{\tau_1=0}=0, \quad (17)$$

and the temperature

$$\frac{\partial\theta}{\partial z}\Big|_{\substack{z=0\\z=l}}=0,\ \theta\Big|_{\tau_1=0}=0,\ \frac{\partial\theta}{\partial\tau_1}\Big|_{\tau_1=0}=0. \quad (18)$$

If we introduce the function Φ

$$u_1=\frac{\partial^2\Phi}{\partial z^2}-\frac{\partial\Phi}{\partial\tau_1}-\beta\frac{\partial^2\Phi}{\partial\tau_1^2},\quad \theta=\gamma\frac{\partial^2\Phi}{\partial z\partial\tau_1}+\gamma\beta\frac{\partial^3\Phi}{\partial z\partial\tau_1^2}, \quad (19)$$

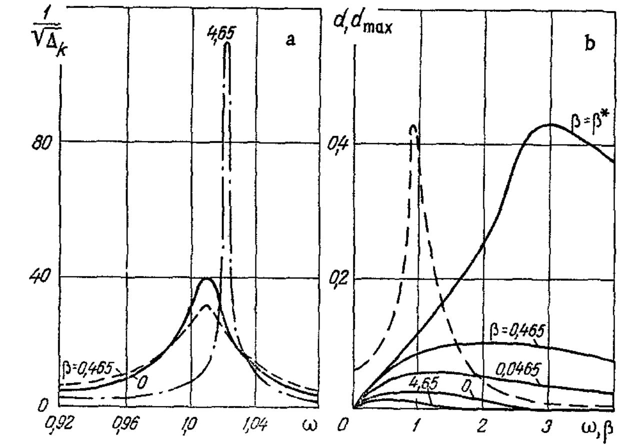

Fig. 4. Quantitative characteristics of thermoelastic rod vibrations.

then the thermoelastic vibrations of the rod under consideration can be described by the equation

$$\frac{\partial^4\Phi}{\partial z^4} - (1+\gamma)\frac{\partial^3\Phi}{\partial z^2\partial\tau_1} - (1+\beta+\gamma\beta)\frac{\partial^4\Phi}{\partial z^2\partial\tau_1^2} + \frac{\partial^3\Phi}{\partial\tau_1^3} + \beta\frac{\partial^4\Phi}{\partial\tau_1^4} = g(z, \tau_1) \tag{20}$$

under appropriate boundary conditions for the function Φ. Representing $\Phi(z, \tau_1)$ in the form of the series

$$\Phi(z, \tau_1) = \frac{2}{l}\sum_{n=1}^{\infty}\Phi_n(\tau_1)\sin\frac{n\pi z}{l}, \tag{21}$$

we obtain an ordinary differential equation for $\Phi_n(\tau_1)$

$$k^4\Phi_n(\tau_1) + \frac{d\Phi_n(\tau_1)}{d\tau_1}k^2(1+\gamma) + \frac{d^2\Phi_n(\tau_1)}{d\tau_1^2}k^2(1+\beta+\gamma\beta) + \frac{d^3\Phi_n(\tau_1)}{d\tau_1^3} + \beta\frac{d^4\Phi_n(\tau_1)}{d\tau_1^4} = g_n\cos\omega\tau_1, \tag{22}$$

where

$$g_0(z) = \frac{2}{l}\sum_{n=1}^{\infty}g_n\sin\frac{n\pi z}{l};\quad g_n = \int_0^l g_0(z)\sin\frac{n\pi z}{l}\,dz. \tag{23}$$

Applying the Laplace transform to (22), we find

$$\bar{\Phi}_n = g_n s/\{(s^2+\omega^2)[\beta s^4 + s^3 + (1+\beta+\gamma\beta)k^2s^2 + k^2(1+\gamma)s + k^4]\}. \tag{24}$$

To perform the inverse Laplace transform, we must investigate the roots of the equation

$$\beta s^4 + s^3 + (1+\beta+\gamma\beta)k^2s^2 + k^2(1+\gamma)s + k^4 = 0. \tag{25}$$

It is easy to show that all the roots of (25) have a negative real part. Upon compliance with the condition $4\beta k^2 > 1$, (25) has two pairs of complex-conjugate roots, while a pair of real roots appears for $4\beta k^2 < 1$.

Using the theorem on decompositions [10], to calculate the original, for $4\beta k^2 > 1$ we find

$$\Phi_n(\tau_1) = \frac{g_n}{\sqrt{\Delta_k}}\sin(\omega\tau_1 + \lambda_k) + g_n\left[\frac{1}{\beta\beta_1}\sqrt{\Delta_1}\exp(-\alpha_1\tau_1)\times\right.$$
$$\left.\times\sin(\beta_1\tau_1 - \lambda_1) + \frac{1}{\beta\beta_2}\sqrt{\Delta_2}\exp(-\alpha_2\tau_1)\sin(\beta_2\tau_1 - \lambda_2)\right], \tag{26}$$

where

$$\Delta_k = [\beta\omega^4 - (1+\beta+\gamma\beta)k^2\omega^2 + k^4]^2 + \omega^2[\omega^2 - k^2(1+\gamma)]^2;$$

$$\Delta_1 = \frac{N^2 + K_1^2}{M^2};\quad \lambda_k = \mathrm{arctg}\,\frac{\beta\omega^4 - (1+\beta+\gamma\beta)k^2\omega^2 + k^4}{\omega[k^2(1+\gamma) - \omega^2]};$$

$$\lambda_1 = \mathrm{arctg}\,\frac{N}{K_1};\quad K_1 = \alpha_1(AC+BD) + \beta_1(BC-AD);$$

$$N = -\alpha_1 (BC - AD) + \beta_1 (AC + BD); \; A = \alpha_1^2 - \beta_1^2 + \omega^2;$$
$$B = 2\alpha_1\beta_1; \; C = (\alpha_2 - \alpha_1)^2 + \beta_2^2 - \beta_1^2; \; D = 2\beta_1 (\alpha_2 - \alpha_1); \tag{27}$$
$$M = (A^2 + B^2)(C^2 + D^2).$$

The expressions for Δ_2 and λ_2 are written analogously to (27).

For $4\beta k^2 < 1$

$$\Phi_n(\tau_1) = \frac{g_n}{\sqrt{\Delta_k}} \sin(\omega\tau_1 + \lambda_k) + g_n \left[\frac{1}{\beta\beta_1} \sqrt{\Delta_3} \exp(-\alpha_1\tau_1) \sin(\beta_1\tau_1 - \lambda_3) \right] +$$

$$+ \frac{a \exp(-a\tau_1)}{(\omega^2 + a^2)[4\beta a^3 - 3a^2 + 2(1 + \beta + \gamma\beta) k^2 a - k^2(1 + \gamma)]} +$$
$$+ \frac{b \exp(-b\tau_1)}{(\omega^2 + b^2)[4\beta b^3 - 3b^2 + 2(1 + \beta + \gamma\beta) k^2 b - k^2(1 + \gamma)]},$$
$$\Delta_3 = \frac{N_3^2 + K_3^2}{M_3^2}, \; \lambda_3 = \operatorname{arctg} \frac{N_3}{K_3}, \tag{28}$$
$$K_3 = \alpha_1 (AC_3 + BD_3) + \beta_1 (BC_3 - AD_3), \; N_3 = -\alpha_1 (BC_3 - AD_3) +$$
$$+ \beta_1 (AC_3 + BD_3), \; M_3 = (A^2 + B^2) \times$$
$$\times (C_3^2 + D_3^2), \; C_3 = (a - \alpha_1)(b - \alpha_1) - \beta_1^2, \; D_3 = \beta_1 (a + b - 2\alpha_1),$$

where $-\alpha_{1,2} \pm i\beta_{1,2}$ or $-a_1$, $-b_1$, $-\alpha_1 \pm i\beta_1$ are the roots of (25), for which analytic expressions can be written by using a series expansion in a small parameter.

Substituting (26) or (28) into (21), we obtain the function $\Phi(z, \tau_1)$, and the values of the displacement and temperature from (19). Let us note that the members in the square brackets in (26) and (28) are the natural thermoelastic vibrations of the rod, which damp out with time. The first members in (26) and (28) describe the forced vibrations of the rod. It follows from (27) that for $\gamma = 0$, $\omega = k$ resonance sets in. For $\gamma \neq 0$, the amplitude of the rod vibrations is bounded at forced vibrations frequencies close to the vibrations frequencies because of thermoelastic energy dissipation. Quantitative characteristics of the thermoelastic rod vibrations are represented in Fig. 4. Values of the dimensionless forced-vibrations amplitude $1/\sqrt{\Delta_k}$ are plotted along the ordinate of Fig. 4a for different β for $k = 1$ and $\gamma = 0.0356$, and the dimensionless frequency ω is plotted along the abscissa. The dependence of the damping decrement $d = 2\pi\alpha_1/\beta_1$ on the frequency (solid lines) for different values of β, as well as the dependence of the maximum values of the decrement d_{max} on β (dashed line) are presented in Fig. 4b. The greatest value of the damping decrement is achieved at $\beta = \beta^*$.

NOTATION

u	is the displacement vector;
σ_{ij}	is the stress tensor component;
e_{ij}	is the strain tensor component;
λ, μ	are the Lame coefficients;
α	is the linear coefficient of temperature expansion;
T	is the temperature;
T_0	is the initial temperature;
$\varkappa$	is the thermal diffusivity;
K	is the coefficient of heat conduction;
c_E	is the specific heat of unit mass;
ρ	is the density;
τ	is the time;
τ_θ	is the heat-flux relaxation time;
τ_f, τ_e	are the phonon and electron relaxation times;
m^*	is the effective mass of the electron;
e	is the charge on the electron;
μ^*	is the mobility;

z_i	is the dimensionless coordinate;
τ_1	is the dimensionless time;
u_1	is the dimensionless displacement;
θ	is the dimensionless temperature;
Σ_{ij}	is the dimensionless stress tensor components;
β	is the dimensionless parameter;
u_0, θ_0	are the wave amplitudes;
p_i, p_R	are the wave numbers;
q_i	is the damping coefficient;
α_n	is the coefficient of heat transfer;
Φ	is some function;
$k = n\pi/l$; l	is the rod length;
A, B, C, D,..., D_p	are the some combinations of parameters introduced;
$\alpha_{1,2}$, $\beta_{1,2}$, a, b	are the roots of the characteristic equation;
v_i	is the phase velocity;
z	is the coordinate along the rod axis.

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